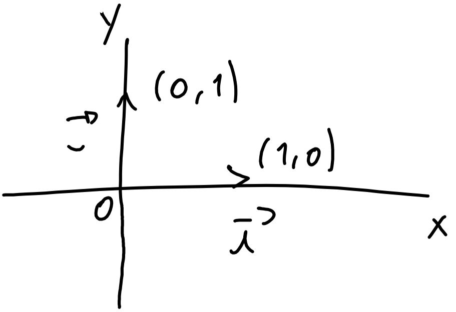
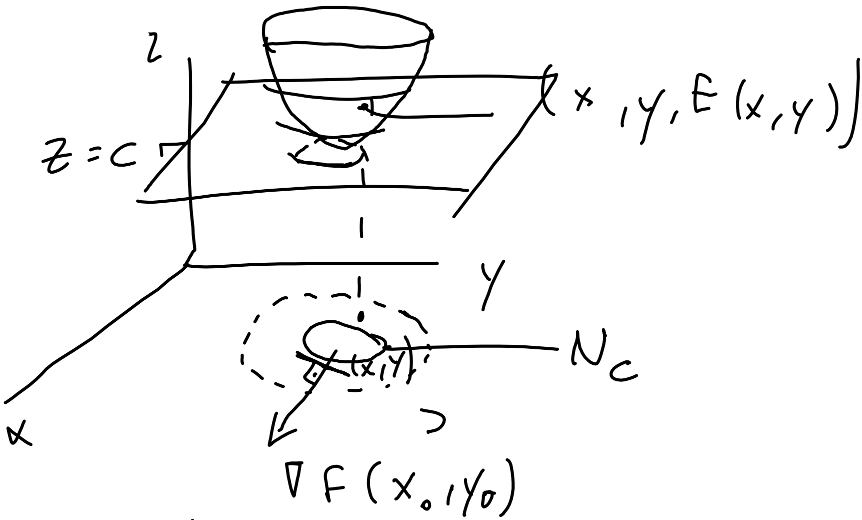
álculo 3 2/3/9 L.1



$$z = f(x,y)$$

$$\Downarrow$$

$$\nabla f(x,y) = \frac{\partial f}{\partial x}\vec{i} + \frac{\partial f}{\partial y}\vec{j} = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right\rangle$$



$$N_c = \{(x,y) \in D : z = c = f(x,y)\}$$

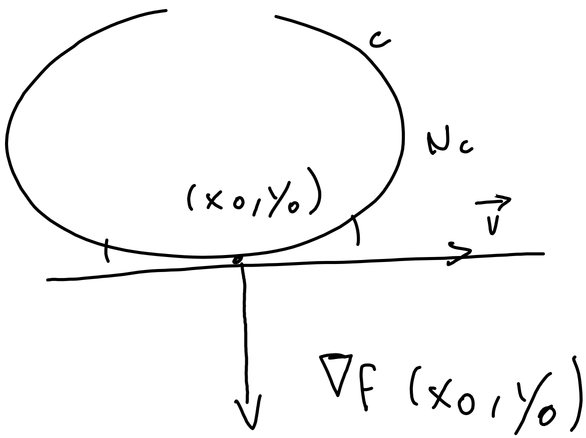
$$\nabla f(x_0,y_0) \perp N_c$$

produto interno é zero

vetor diretor da reta tangente

gradiente

$$\langle \vec{v}, \nabla f(x_0, y_0) \rangle$$



$$\vec{\gamma}(t) = h(t)\vec{i} + g(t)\vec{j}$$

$t \in I$

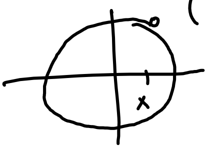
$$\vec{\gamma}(t_0) = (x_0, y_0)$$

$$\vec{v} = \vec{\gamma}\,'(t_0)$$

L.3

$$\begin{cases} y = P(x) \Rightarrow \vec{\gamma}(x) = (x, P(x)) \\ x \in J(x_0) \end{cases}$$

$$\boxed{\vec{\gamma}\,'(x) = (1, P'(x))}$$



(x_0, y_0) $x^2 + y^2 = 1$

$y = \varphi(x)$

Teorema: O gradiente é normal às curvas de nível

"Seja a superfície $z = f(x,y)$ e considere $N_c \neq \emptyset = \{(x,y) : f(x,y) = c\}$

Se para $(x_0, y_0) \in N_c$, $\nabla f(x_0, y_0) \neq \vec{0}$

$\Rightarrow \nabla f(x_0, y_0) \perp N_c$ no ponto (x_0, y_0)

Prova: suponha que perto de (x_0, y_0), o gráfico de N_c é o gráfico de uma função $y = P(x)$, $x \in J(x_0)$

$$\underline{f(x, P(x)) = C} \quad \forall x \in J(x_0)$$

$$\frac{d}{dx} f\big(x, \overbrace{\cancel{g}\,P(x)}^{y}\big) = 0$$

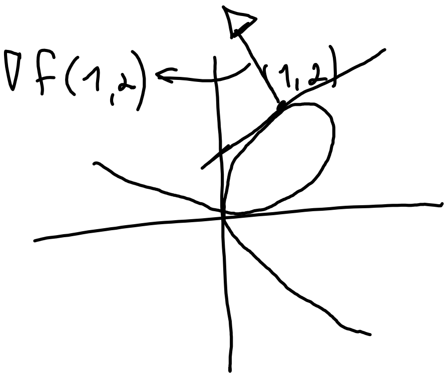
$$\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}\frac{dP}{dx} = 0$$

$$\left\langle \nabla f(x_0, y_0), \left(1, \frac{dP(x_0)}{dx}\right) \right\rangle = 0$$

$$\langle \nabla f(x_0, y_0), \vec{\gamma}\,'(t_0) \rangle = 0$$

$$\nabla f(x_0, y_0) \perp \vec{\gamma}\,'(t_0)$$

Exemplo: Escrever a equação da reta tangente, no ponto $(1,2)$ ao folium de Descartes da eq $2x^3 + 2y^3 - 9xy = 0$



115

$$z = f(x,y)$$

$$N_0 = \{(x,y) : f(x,y) = 0\}$$

$$\nabla f(x,y) = \langle 6x^2 - 9y,\ 6y^2 - 9x \rangle$$

$$\nabla f(1,2) = \langle -12,\ 15 \rangle$$

$$(x,y) \in L \Rightarrow (x,y) - (1,2) = (x-1, y-2) = \vec{v}$$

$$\nabla f(1,2) \cdot \vec{v} = 0$$

$$\langle -12, 15 \rangle \cdot \langle x-1, y-2 \rangle = 0$$

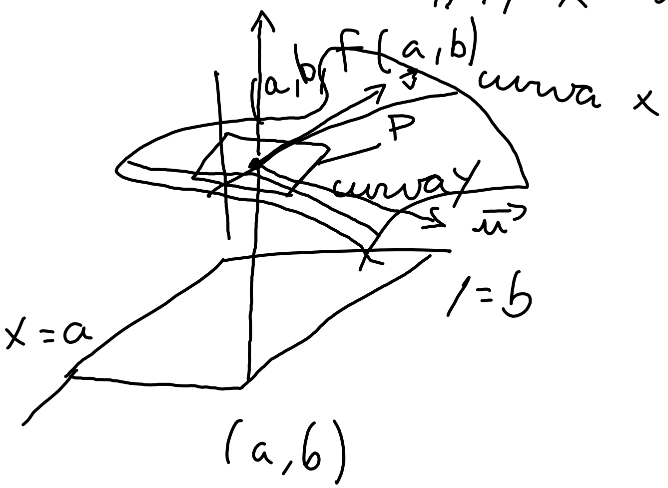
$$-12(x-1) + 15(y-2) = 0$$

$$\boxed{4x - 5y + 6 = 0}$$

Plano Tangente a $z = f(x,y)$

Suponha que f_x, f_y são contínuas em $R(a,b)$. O plano tangente à superfície $z = f(x,y)$ no ponto $P = (a, b, f(a,b))$ é o plano por P que contem as retas tangentes às duas curvas

$$
*)\ \begin{cases} z = f(x,y),\ y = b & (\text{curva } x) \\ z = f(x,y),\ x = a & (\text{curva } y) \end{cases}
$$



$$
\langle (x-a), (y-b), z - f(a,b) \rangle \cdot \vec{n} = 0
$$

$\langle a, y, f(a,y) \rangle = \vec{\gamma}(y)$

$\langle 0, 1, \frac{\partial f}{\partial y}(a,y) \rangle = \vec{\gamma}'(y)$

$\vec{u} = \vec{\gamma}'(b) = \vec{j} + \frac{\partial f}{\partial y}(a,b)\vec{k}$

urwa x

$\vec{\smile}(x) = \langle x, b, f(x,b) \rangle$

$\vec{\smile}'(x) = \langle 1, 0, \frac{\partial f}{\partial x}(x,b) \rangle$

$\vec{V} = \vec{S}'(a) = \langle 1, 0, \frac{\partial f}{\partial x}(a,b) \rangle$

$$\vec{n} = \vec{u} \times \vec{V} = \begin{vmatrix} i & j & k \\ 0 & 1 & \frac{\partial f}{\partial y}(a,b) \\ 1 & 0 & \frac{\partial f}{\partial x}(a,b) \end{vmatrix}$$

$\vec{n} = \left(\frac{\partial f}{\partial x}(a,b)\right)\vec{i} + \left(\frac{\partial f}{\partial y}(a,b)\right)\vec{J} +$

$+ (-1)\vec{k} =$

$$\boxed{\left\langle \frac{\partial f}{\partial x}(a,b), \frac{\partial f}{\partial y}(a,b), -1 \right\rangle = \vec{n}}$$

Eq. p: $\dfrac{\partial f}{\partial x}(a,b)(x-a) + \dfrac{\partial f}{\partial y}(a,b)(y-b) -$

$- (z - f(a,b)) = 0$

Exemplo: eq. do plano tangente ao paraboloíde $z = x^2 + y^2$ no ponto $P = (2,-1,5)$

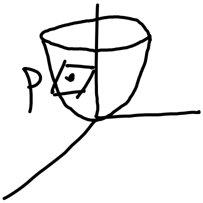
$\vec{n} = (\nabla f(2,-1), -1)$

$= (4,-2,-1)$

$\left(\dfrac{\partial f}{\partial x}(2,1), \dfrac{\partial f}{\partial y}(2,-1), -1 \right)$

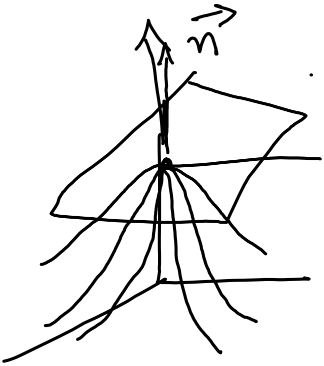
$4(x-2) - 2(y+1) - (z-5) = 0$

$4x - 2y - z = 5$



$$z = f(x,y) = e^{-x^2-y^2} \qquad P = (0,0)$$

plano tangente



$z \leq 1$

$(0,0,1)$ $\vec{n} = (0,0,-1)$

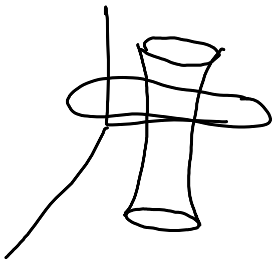
$$\frac{\partial f}{\partial x} = -2x e^{-x^2-y^2}$$

$$\frac{\partial f}{\partial y} = -2y e^{-x^2-y^2}$$

$$v = F(x,y,z) = f(x,y) - z$$

$$\nabla F(x,y,z) = \left\langle \frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}, -4 \right\rangle \qquad \angle 0$$

$$\nabla F(0,0,1) = \vec{n}$$



$$F(x,y,z) = c$$

$$\nabla F \perp S_c$$