Cálc. III

.6/2 [illegible]

Até quarta revisão

Transformações de $\mathbb{R}^n$ em $\mathbb{R}^m$

$\mathbb{R}^m = \{ (x_1, \ldots, x_m) : x_i \in \mathbb{R} \}$

Cursos anteriores: 1 - $T: \mathbb{R} \to \mathbb{R}$ fç real de variável real

2 - $T: \mathbb{R} \to \mathbb{R}^m$, $m > 1$, fç vetorial de uma variável real

3 - $T: \mathbb{R}^n \to \mathbb{R}$, $n \geq 2$, fç real de uma variável vetorial (campo escalar)

4 - $T: \mathbb{R}^n \to \mathbb{R}^m$, $m \geq 2$, fç vetorial de variável vetorial.

$T: D \subseteq \mathbb{R}^2 \to \underline{\underline{\mathbb{R}}}$

D um conjunto aberto

$\to B_\epsilon(p) \subseteq D$

1 – T é contínua em $(a,b) \in D$

i – $\lim\limits_{(x,y)\to(a,b)} T(x,y) = L < \infty$

ii) $T(a,b) = L$

Exemplo: a) $f(x,y) = \text{sen}(x+y)$

Contínua $\forall (a,b) \in \mathbb{R}^2$

$$\lim_{(x,y)\to(a,b)} \text{sen}(x+y) = \text{sen}(a+b)$$

b) $f(x,y) = \begin{cases} \dfrac{5x^2y}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$

Cálc III

a? para f contínua em (0,0)

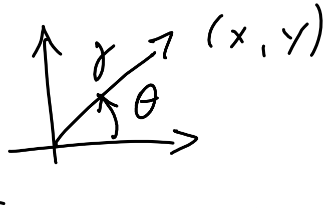
$$\lim_{(x,y)\to(0,0)} \frac{5x^2y}{x^2+y^2} = 0$$

$$0 \le \left|\frac{5x^2y}{x^2+y^2}\right| \le \frac{5x^2}{x^2+y^2}|y| \le 5|y| \xrightarrow{(x,y)\to(0,0)} 0$$

c) $$f(x,y) = \begin{cases} \dfrac{\operatorname{sen}(x^2+y^2)}{x^2+y^2} & (x,y)\neq(0,0) \\ a & (x,y)=(0,0) \end{cases}$$

$x = r\cos\theta$

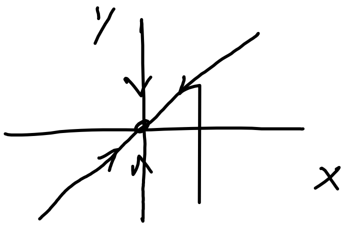
$y = r\operatorname{sen}\theta$



$f(x,y) = \dfrac{\operatorname{sen} r^2}{r^2}$, $\lim_{(x,y)\to(0,0)} f(x,y) =$

$$= \lim_{r\to 0} \frac{\operatorname{sen} r^2}{r^2} = 1 \quad \{ (x,y)\to(0,0), r\to 0$$

$$\lim_{(x,y)\to(0,0)} \left(\frac{x^2-y^2}{x^2+y^2}\right)^2 = \begin{cases} 1 & \text{eixo } y \\ 0 & \text{na reta } y = x \end{cases}$$



$f: D \subseteq \mathbb{R}^2 \to \mathbb{R}$, $(a,b) \in D$

$$\frac{\partial f}{\partial x}(a,b) = \lim_{h\to 0} \frac{f(a+h,b)\ f(a,b)}{h}$$

$$\frac{\partial f}{\partial y}(a,b) = \lim_{h\to 0} \frac{f(a,b+h) - f(a,b)}{h}$$

Notações: $z = f(x,y)$

$$z_x = f_x = \frac{\partial f}{\partial x}$$

$$z_y = f_y = \frac{\partial f}{\partial y}$$

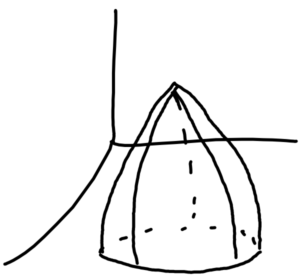
interpretação geométrica de $\dfrac{\partial f}{\partial x}(a,b)$ *

$x \to f(x,b)$

$z = f(x,y)$, $G(f) = \{(x,y,z) : z = f(x,y), (x,y) \in D\}$

* inclinação da tangente à curva, no plano $y = b$

Exemplo: Calcular as inclinações na superfície $f(x,y) = 1-(x-1)^2-(y-2)^2$ no ponto $(1,2,1)$



$$\frac{\partial f}{\partial x}(1,2) = -2(x-1)\Big|_{x=1} = 0$$

$$\frac{\partial f}{\partial y}(1,2) = -2(y-2)\Big|_{y=2} = 0$$

$$x \to f(x, 2) = 1 - (x-1)^2$$

> Derivadas parciais de ordem superior

$$\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial x^2}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial y \partial x}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 F}{\partial x \partial y}$$

Teo. Schwarz

$$\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial y^2}$$

Exemplo: $f(x, y) = \operatorname{sen}(xy) + e^{x^2 + y^2}$

$$\frac{\partial f}{\partial x} = x \cos(xy) + 2y\, e^{x^2 + y^2}$$

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Teorema (Schwarz)

$f: D \subseteq \mathbb{R}^2 \to \mathbb{R}$. Suponha f_{xy}, f_{yx} são contínuas em $D \Rightarrow f_{xy}(a,b) = f_{yx}(a,b)$

Equação de Laplace

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0 \quad z = f(x,y)$$

$$z = \frac{1}{2}\left(e^{y} - e^{-y}\right) \operatorname{sen} x$$

Equação de onda

$$z = f(x,t) \quad \frac{\partial^2 z}{\partial t^2} = c^2 \left(\frac{\partial^2 z}{\partial x^2}\right)$$

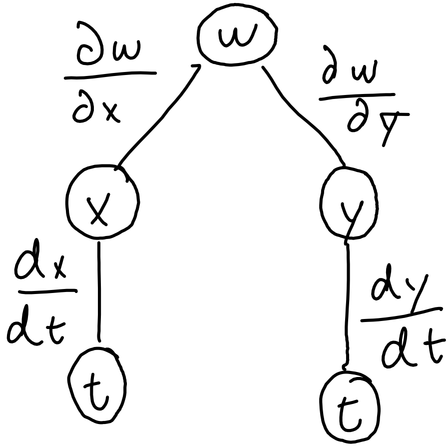
$$z = \ln(x + ct)$$

Regra da Cadeia

1 - Uma variável independente

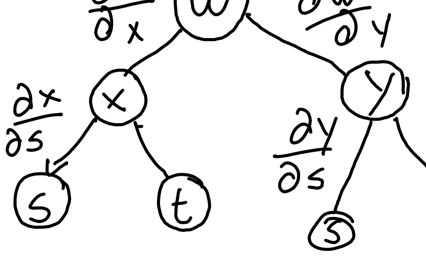
$$w = f(x,y), \quad x = g(t), \quad y = h(t)$$

$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dg}{dt} + \frac{\partial w}{\partial y}\frac{dh}{dt}$$



2 - Duas variáveis independentes

$$w = f(x,y), \quad x = g(s,t), \quad y = h(s,t)$$



$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x}\frac{\partial g}{\partial s} + \frac{\partial w}{\partial y}\frac{\partial h}{\partial s}$$

$$\frac{\partial w}{\partial t}$$

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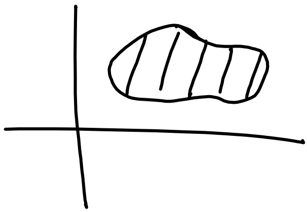
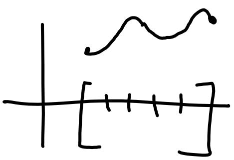
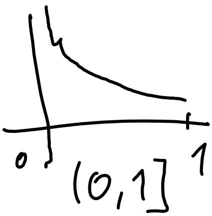
Extremos de funções de duas variáveis

1- Seja $f : R \subseteq \mathbb{R}^2 \longrightarrow \mathbb{R}$

R região fechada/limitada







1- $(a,b) \in R$ é chamado mínimo de F em $\mathbb{R}$ se $f(x,y) \geq f(a,b) \; \forall (x,y) \in \mathbb{R}$

2- $(a,b) \in \mathbb{R}$ é chamado máximo de F em $\mathbb{R}$ se $f(x,y) \leq f(a,b) \; \forall (x,y) \in \mathbb{R}$

Cálculo III 16/2/17

Teorema: Sup. $f: R \subseteq \mathbb{R}^2 \to \mathbb{R}$ Cont. em R

1. $\exists$ pelo menor um ponto em R onde f assume um valor mínimo

2. $\exists$ pelo menor um ponto em R onde f assume um valor máximo