(Nyquist)

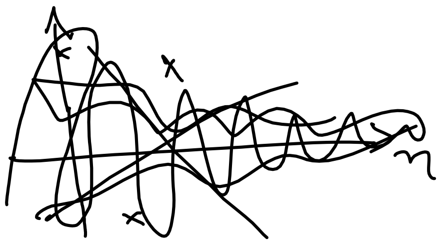
Um sinal digital amostrado a R Hz só pode representar corretamente freqüências de até $\frac{R}{2}$ Hz

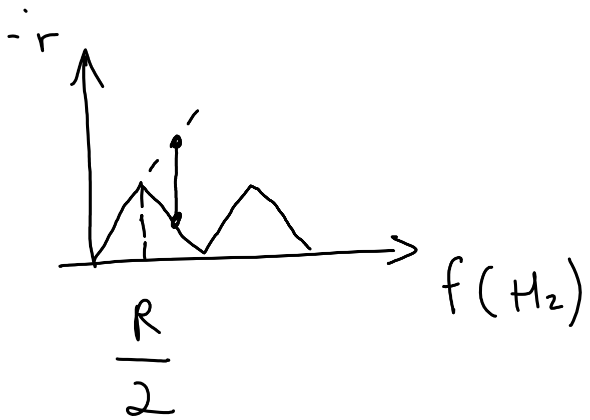
$$\text{sen}(wn) \equiv \text{sen}((w+2\pi)n) \quad \forall k \in \mathbb{Z} \quad \forall n$$

$$\text{sen}(wn) \equiv \text{sen}\left(2\pi(f+kR)\frac{n}{R}\right)$$

$\{f+kR\}_{k\in\mathbb{Z}}$ são representados do mesmo modo.

$$\frac{R}{2} \leq f_r \leq +\frac{R}{2}$$





$$-\frac{R}{2} \leq f + kR \leq \frac{R}{2}$$

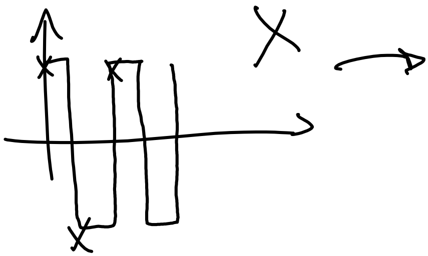
$$-1/2 \leq \frac{f}{R} + k \leq 1/2$$

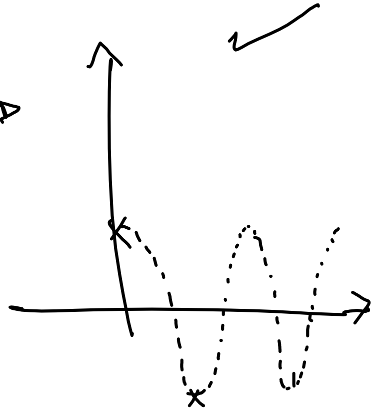
$$\boxed{-\frac{1}{2} - \frac{f}{R} \leq k \leq \frac{1}{2} - \frac{f}{R}}$$

$$k = \text{round}\left(-\frac{f}{R}\right)$$

$$f_r = f + kR$$

Exemplo.



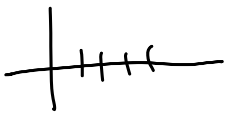


$$F_0 = \frac{R}{2}\ \text{Hz}$$

20

1 harmônico de $\frac{3R}{2}$ seria uma senóide

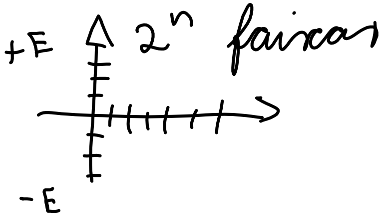
- DC



↓

Quantização

↓



Quantização linear

n bits

2^n faixas, cada uma de tamanho $\frac{2E}{2^n}$

diferença entre valor real e quantizado:

ruído de quantização $\leq \frac{E}{2^n}$

$$SQNR = \frac{\text{nível do sinal}}{\text{pior nível do ruído}} \leq \frac{E}{E/2^n} = 2^n$$

$$SQNR(dB) \leq 10 \log_{10} \left(\frac{E}{E/2^n}\right)^2 =$$

→ pressão p/ intensidade

$$20 \underbrace{\log_{10} 2}_{\cong 0{,}3}{}^{n} \cong 6N$$

$$SQNR(dB) = 20 \log_{10} \frac{\text{amp sinal}}{\text{amp ruído}}$$

$$= 20 \log_{10} \cancel{} \cdot \text{amp } \cancel{\text{sinal}}$$

$$= 20 \log_{10} \frac{\text{amp sinal}}{\text{amp max}} \cdot \frac{\text{amp max}}{\text{amp ruído}}$$

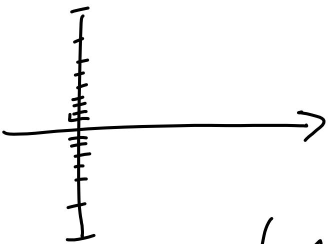
$$= 20 \log_{10} \frac{''}{''} + 20 \log_{10} \frac{''}{''}$$

$$= 6N + S$$

$$\frac{E/2}{E} = 1/2$$

$$20 \log_{10} 1/2 = 20 \log_{10} 2^{-1} = -20 \log_{10} 2 = -6$$

Quantização não linear



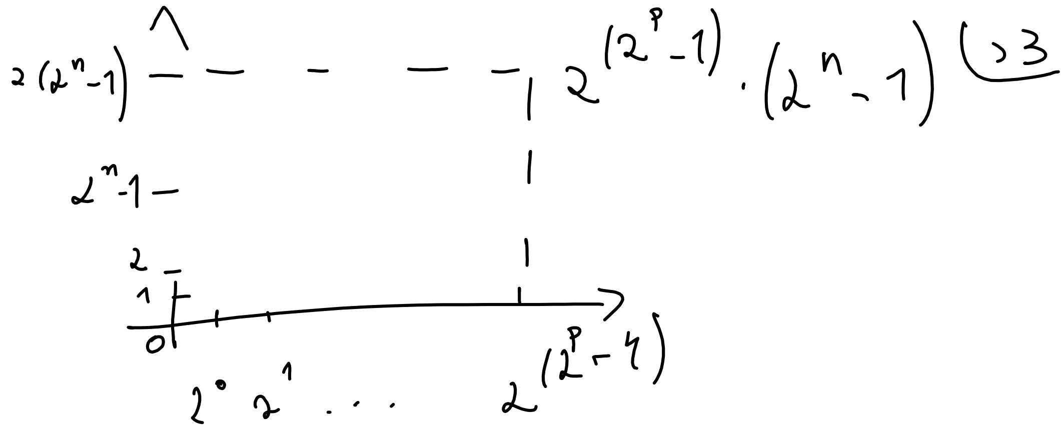
Quantização logarítmica

$\boxed{\mu\text{-LAN}}$

$$S_\mu = \log_{256} (1 + 255 s)$$

Quantização em ponto flutuante

$$\underbrace{\text{expoente}}_{p \text{ bits}} \quad \underbrace{\text{código}}_{n \text{ bits}}$$



amp. max. representável = $2^{(2^P-1)} \cdot (2^n-1)$

amp. min. representável = 1

amp. ruído. quantização depende ~~da faixa~~ da amplitude

melhor caso: 1/2

pior caso: $2^{(2^P-2)}$

faixa dinâmica = $2^{(2^P-1)}(2^n-1)$

$$\simeq (2^P-1)6 + n6 = 6(n + 2^P - 1)$$

$$\frac{S}{1/2} \le SQNR \le \frac{S}{2^{(2^P-2)}}$$

$$20\log_{10} \frac{S}{\text{amp. max}} \cdot \frac{\text{amp max}}{1/2} \le SQNR\,(dB) \le$$

$$\leq 20 \log_{10} \frac{S}{\text{amp. max}} \cdot \frac{\text{amp max}}{2^{(2^p-2)}}$$

$$S_{dB} + 6\left(2^p + n\right) \overset{\times}{\leq} SQNR(dB) \overset{\times}{\leq} S_{dB} + 6(n+1)$$

$$\geq \qquad\qquad \geq$$

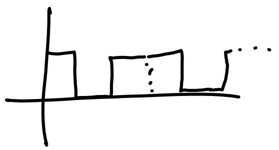
— // —

Codificação:

PCM: pulse code modulation

amostra → $\underline{01100}$ · sinais, menor erro

código binario



+ divisões de volts

· PAM: pulse-amplitude modulation

PWM: pulse width modulation

DPCM: differencial PCM

transmite apenas os saltos

ADPCM: adaptação conforme o tamanho ~~dos diferen~~ dos saltos

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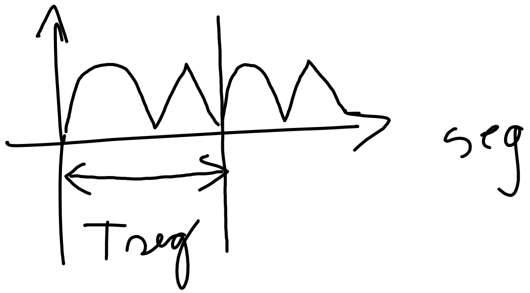
Funções Periodicas

Def: $f(\cdot)$ é periodica

se $\exists\, T > 0$ t.q.

$$f(x+T) = f(x) \ \forall x$$



Se f é periódica c/ período $T > 0$ ela pode ser representada por uma família de senóides c/ períodos $T/1, T/2, T/3, T/4, \dots$ (frequências $f_0 = 1/T$, $2f_0$, $3f_0$, ...)

Vamos considerar a família de senóides $A \operatorname{sen}(w_i t + \varphi)$

$f_0 = \frac{1}{T}$, $f_i = i \cdot f_0$, $w_0 = 2\pi f_0$, $w_i = i w_0 = i 2\pi f_0$

Relações Trigonométricas Importantes

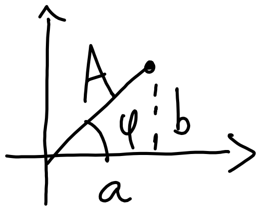
$$\operatorname{sen}(a+b) = \operatorname{sen}(a)\cos(b) + \operatorname{sen}(b)\cos(a)$$

$$A \operatorname{sen}(w_i t + \varphi) = A\left[\operatorname{sen}(w_i t)\cos(\varphi) + \operatorname{sen}(\varphi)\cos(w_i t)\right]$$

$$= \underbrace{A\cos(\varphi)}_{a} \operatorname{sen}(w_i t) + \underbrace{A \operatorname{sen}(\varphi)}_{b} \cos(w_i t)$$

$= a \operatorname{sen}(w_i t) + b \cos(w_i t)$ onde

$$a = A\cos\varphi \qquad A = \sqrt{a^2 + b^2}$$
$$b = A \operatorname{sen}\varphi \qquad \varphi = \operatorname{arctg} b/a$$



Lema (Ortogonalidade) → de senos e cossenos

$$\int_0^T \operatorname{sen}(w_i t)\cos(w_j \cdot t)\,dt = 0 \quad \forall\, i, j$$

$$\int_0^T \cos(w_i t)\cos(w_j t)\,dt = \begin{cases} 0, & \text{se } i \neq j \\ T, & \text{se } i = j = 0 \\ T/2, & \text{se } i = j > 0 \end{cases}$$

$$\int_0^T \operatorname{sen}(w_i t)\operatorname{sen}(w_i t)\,dt = \begin{cases} 0 & \text{se } i \neq j \\ T & \text{se } i = j = 0 \\ T/2 & \text{se } i = j > 0 \end{cases}$$

Prova

(i) . . .

]

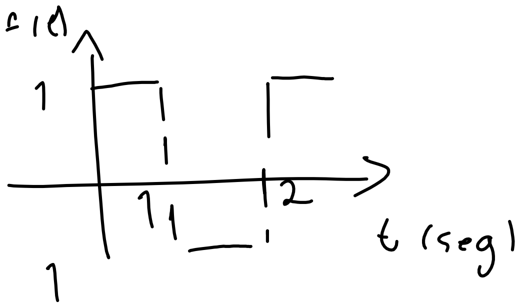
∴ ———

$$\operatorname{sen}(x)\cos(y) = \frac{1}{2}\left[\operatorname{sen}(x+y) + \operatorname{sen}(x-y)\right]$$

, ———

$$\cos(x)\cos(y) = \frac{1}{2}\left[\cos(x+y) + \cos(x-y)\right]$$

Exemplo:



$T = 2\,seg$

$F = 0{,}5\,Hz$

$w = 2\pi\, 0.5 = \pi$

Usaremos a família de senóides

$$A_i \operatorname{sen}(i \omega t + \varphi)$$

$$i \pi t$$

— // —

$$a_j = \frac{2}{T} \int_0^T f(t) \operatorname{sen}(\omega_j t)\, dt, \quad j \neq 0$$

$$b_j = \begin{cases} \dfrac{1}{T} \displaystyle\int_0^T f(t) \cancel{\cos(\omega_j t)}\, dt, & \text{se } j = 0 \\ \dfrac{2}{T} \displaystyle\int_0^T f(t) \cos(\omega_j t)\, dt, & \text{se } j > 0 \end{cases}$$

$$f(x) = \sum_{i=0}^{\infty} a_i \operatorname{sen}(\omega_i t) + b_i \cos(\omega_i t)$$

// —

> $j > 0$

$$a_j = \frac{2}{T} \int_0^T f(t)\, \operatorname{sen}(w_j t)\, dt =$$

$$= \int_0^1 \operatorname{sen}(w_j t)\, dt - \int_1^2 \operatorname{sen}(w_j t)\, dt$$

$$= \left[\frac{-\cos(w_j) + \cos(0)}{w_j}\right] - \left[\frac{-\cos(2w_j) + \cos(w_j)}{w_j}\right]$$

$$\frac{-\cos(0) - 2\cos(w_j) + \cos(2w_j)}{w_j}$$

$$= \frac{\overbrace{\cos(0)}^{-1} - 2\cos(j\pi) + \overbrace{\cos(j2\pi)}^{1}}{j\pi}$$

$$= \begin{cases} 0 & \text{se } j \text{ é par} \\ \dfrac{4}{j\pi} & \text{se } j \text{ é impar} \end{cases}$$

$$b_0 = \frac{1}{T}\int_0^T f(t)\cos(0t)\,dt$$

$$= \frac{1}{2}\int_0^T f(t)\,dt = 0$$

$$b_j = \frac{2}{T}\int_0^T f(t)\cos(w_j t)\,dt$$

$$= \int_0^1 \cos(w_j t)\,dt + \int_0^2 \cos(w_j t)\,dt$$

$$= \left[\frac{\operatorname{sen}(w_j) - \operatorname{sen}(0)}{w_j}\right] + \left[\frac{\operatorname{sen}(2w_j) - \operatorname{sen}(w_j)}{w_j}\right]$$

$$= 0, \quad j > 0$$