24/4/2008 (132)

MAC0315 - Prog Lin - Exercícios 3

1. $\Rightarrow$ $\exists$ n restrições de P ativas L_i e as demais restrições são satisfeitas por desigualdade

$Ax^* + b - Ax^* = b$, $x^* \geq 0$, $b - Ax^* \geq 0$

$(x^*, b - Ax^*)$ básica viável não degenerada.

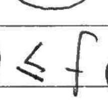
$\Delta =$

$z \geq 0$ $\quad (x^*, b - Ax^*)$?

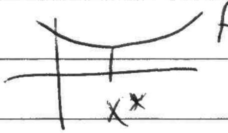
$\Leftrightarrow$ "$b \geq Ax$"

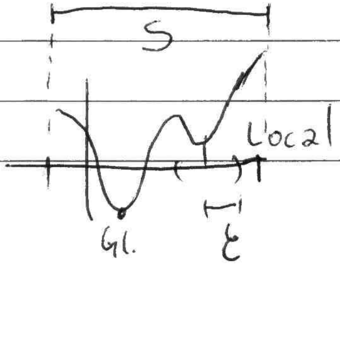
2. x^* é min local de P $\Leftrightarrow$ x^* é min global de P. Def. x^* é min local de P se $\exists\, \varepsilon > 0$ tq $f(x^*) \leq f(x)$, $\forall x \in S \cap B(x^*, \varepsilon) =$

$= \{x \in R^n \; \|x - x^*\| < \varepsilon$



x^* é min global se $f(x^*) \leq f(x)$, $\forall x \in S$

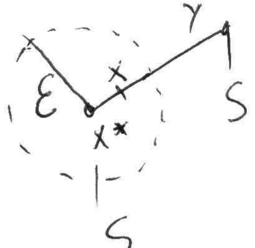




(⇐) trivial

(⇒) Basta provar que $f(x^*) \leq f(y), \forall y \in S \setminus B(x^*, \varepsilon)$

Tome $y \in S \setminus B(x^*, \varepsilon)$. $\exists x \in B(x^*, \varepsilon)$ tq



$x = \lambda x^* + (1-\lambda) y$, para algum $\lambda \in]0,1[$ $\in S$ pois S é convexo

$f(x^*) \leq f(x) = f(\lambda x^* + (1-\lambda) y)$
$\leq \lambda f(x^*) + (1-\lambda) f(y)$ [pois f é convexa]

$\Rightarrow f(x^*) \leq f(y)$

Logo, x^* é min global

3. $\exists \theta > 0$ tq $x + \theta d \in P$

(⇒) $\exists \theta \geq 0$ tq $x + \theta d \in P$

$A(x+\theta d) = b \Rightarrow \theta A d = 0 \Rightarrow Ad = 0$

$\left.\begin{matrix} x + \theta d \geq 0 \\ x_i = 0 \end{matrix}\right\} \Rightarrow \theta d_i \geq 0 \Rightarrow d_i \geq 0$

(⇐) Seja $\theta =$ ______ > 0

$A(x + \theta d) = Ax = b$

$x_i = 0$: $x_i + \theta d_i = \theta d_i \geq 0$

$x_i \neq 0$ e $d_i = 0$: $x_i + \theta d_i = x_i \geq 0$

$x_i \neq 0$ e $d_i > 0$: $x_i + \theta d_i > 0$

$x_i \neq 0$ e $d_i < 0$: $\underbrace{x_i}_{\geq 0} + \underbrace{\theta d_i}_{< 0} \overset{?}{\geq} 0$

$$x_i - \underbrace{\frac{x_i}{d_i}}_{\theta} d_i$$

$$\theta = \max_{\substack{1 \leq i < n \\ d_i < 0}} \left\{ -\frac{x_i}{d_i} \right\} > 0$$

$$x_i + \theta d_i = x_i + \max_{\substack{1 \leq j \leq n \\ d_j < 0}} \left\{ -\frac{x_j}{d_j} \right\} d_i$$

$$\geq x_i - \frac{x_i}{d_i} d_i = 0$$

4. $\min c'x$ (1)

SA $Ax = b$

$x \geq 0$

(2)

$\min c'd$

SA $Ad = 0$

$d_i \geq 0, \ \forall i \in Z = \{i : \bar{x}_i = 0\}$

$\bar{x}$ é sol. viável de (1)

$\bar{x}$ é ótima de (1) $\iff$ 0 é ótimo de (2)

$[c'\bar{x} \leq c'x, \ \forall x \in P]$ $[c'd \geq 0, \ \forall d \in Q]$

($\Leftarrow$) Seja $x \in P$ e defina $d = x - \bar{x}$

$$\left.\begin{array}{l} Ad = A(x - \bar{x}) = 0 \\ \text{Se } \bar{x}_i = 0 \Rightarrow d_i = x_i - \bar{x}_i = x_i \geq 0 \end{array}\right\} \Rightarrow d \in Q$$

$c'd \geq 0 \Rightarrow c'(x-\bar{x}) \geq 0 \Rightarrow c'\bar{x} \leq c'x$ 135

$(\Rightarrow)$ Seja $d \in Q$ e defina $x = \bar{x} + \theta d$

$$\theta = \max_{\substack{1 \leq k \leq n \\ d_k < 0}} \left\{ -\frac{\bar{x}_k}{d_k} \right\}$$

$$Ax = A(\bar{x} + \theta d) = A\bar{x} + \theta \cancel{Ad}^{0} = b$$

$\bar{x}_i = 0$: $x_i = \bar{x}_i + \theta d_i = \theta d_i \geq 0$

$$\bar{x}_i \neq 0 : x_i = \bar{x}_i + \max_{\substack{1 \leq k \leq n \\ d_k < 0}} \left\{ -\frac{\bar{x}_k}{d_k} \right\} d_i \begin{cases} \geq \\ = \end{cases}$$

$\geq \bar{x}_i - \frac{\bar{x}_i}{d_i} d_i = 0$ se $d_i \neq 0$

$= \bar{x}_i \geq 0$ se $d_i = 0$

$\Rightarrow x \in P$

$\Rightarrow c'\bar{x} \leq c'x \Rightarrow c'\bar{x} \leq c'(\bar{x} + \theta d) \Rightarrow \theta c'd \geq 0 \Rightarrow c'd \geq 0.$

5. $\min c'x$ custo ótimo $= -\infty \Leftrightarrow \exists d \neq 0$ tq $d \geq 0$. $Ad \geq 0$. $c'd < 0$

SA $Ax \geq b$

$x \geq 0$

(poliedro é ilimitado)

$(\Leftarrow)$ Seja $x \in P$, $\alpha > 0$

$A(x + \alpha d) = Ax + \alpha Ad \geq b$

$\cancel{A}x \ \ldots$
$x + \alpha d \geq 0$ $\Big\}$ $\Rightarrow x + \alpha d \in P, \ \forall \alpha > 0$

$c'(x + \alpha d) = c'x + \alpha \underbrace{c'd}_{<0} \longrightarrow -\infty$ se

$\alpha \longrightarrow \infty$

($\Rightarrow$) custo ótimo $= -\infty \Rightarrow \exists d \neq 0$ tq
$x + \alpha d \in P$ e $c'(x + \alpha d) \longrightarrow -\infty$ quando
$\alpha \xleftarrow{\alpha > 0} \infty$, $\forall x \in P$

$c'(x + \alpha d) = c'x + \alpha c'd \to -\infty \Rightarrow \alpha c'd \xrightarrow{\alpha \to \infty} -\infty$
$\alpha \geq 0$

$\Rightarrow c'd < 0$

$A(x + \alpha d) = Ax + \alpha A d \geq b$
$Ax \geq b$ $\Big\}$ $\Rightarrow \cancel{\alpha} Ad \geq 0$ pois $\alpha \to \infty$

$x + \alpha d \geq 0$
$x \geq 0$ $\Big\}$ $\Rightarrow d \geq 0$ pois $\alpha \to \infty$