Formais 19/11 | 54

1. Usando a linguagem vista em aula, definir repeat {C} until B.

2. Provar:

a $\vdash_{par} (| x > 0 |)\ y = x - 1\ (| y > 1 |)$

b $\vdash_{par} (| \top |)\ y = x;\ y = x + x + y;\ (| y = 3x |)$

c $\vdash_{par} (| x > 1 |)\ a = 1;\ y = x;\ y = y - a;\ (| y > 0 \wedge x > y |)$

3. Escreva P e prove:

$\vdash_{par} (| \top |)\ P\ (| z = \max(w, x, y) |)$

4. Dado o programa

```
mult : z = 0;
       while (y != 0) {
           z = z + x
           y = y - 1;
       }
```

Prove LSS

(a) $\vdash$ par $(|y = y_0 \wedge y \geq 0|)$ mult $(|z = x \cdot y_0|)$

(b) $\vdash_{TOT}$ //

——— // ———

Respostas

1 C;
while (!B) {
    C;
}

2. a) $(|x > 0|)$
$(|x + 1 > 1|)$ Impl
$y = x + 1;$
$(|y > 1|)$ atrib

b) $(|T|)$

$(|x + x + x = 3x|)$ Impl

$y = x;$

$(|x + x + y = 3x|)$ atrib

$y = x + x + y;$

$(|y = 3x|)$ atrib

c) $(|x > 1|)$

$(|x - 1 > 0 \wedge x > x - 1|)$ Impl

$a = 1$

$(|x - a > 0 \wedge x > x - a|)$ atrib

$y = x;$

$(|y - a > 0 \wedge x > y - a|)$ atrib

$y = y - a;$

$(|y > 0 \wedge x > y|)$ atrib

(3) P: if (w > x) { $(| w > x \to \varphi_1 \wedge \neg(w > x) \to \varphi_2 |)$ [5 7

if (y > w) { $(| y > w \to y = max \wedge \neg(y > w) \to w = max |) = \varphi_1$

$(| y = max |)$ if

z = y;

$(| z = max |)$ atrib

}

else

$(| w = max |)$ else

z = w;

$(| z = max |)$ atrib

}

else {

$(| y > x \to y = max \wedge \neg(y > x) \to x = max |) = \varphi_2$

if (y > x) {

$(| y = max |)$ if

z = y;

$(| y = max |)$ atrib

}

else z = x;

$(| z = max |)$ atr.

}

$(| z = max(w, x, y) |)$ if else

LS8

$\vdash (w > x \to (y > w \to y = max) \land$
$\neg(y > w) \to w = max)) \land$
$(\neg(w > x) \to (y > x \to y = max \land$
$\neg(y > x) \to x = max))$

1)

1. $w > x$ sup
2. $y > w$ sup
3. $y = max$
4. $y > w \to y = max$ $\to_i$
5. $\neg(y > w)$ sup
6. $w = max$
7. $\neg(y > w) \to w = max$ $\to_i$
8. $(y > w \to y = max) \land \neg(y > w) \to w = max$ $\land_i$

```
while (x != 1) {
    if (x % 2 == 0)
        x = x/2;
    else x = 3 * x + 1;
}
```

Sem prova que termina. Dificuldade em achar o invariante.

$(| 10 = x(y_0 - y) |) \uparrow (| y = y_0 \wedge y \geq 0 |)$ $\wedge\, 0 \leq y |)$

$z = 0$

$(| z = x(y_0 - y) |) \wedge 0 \leq y |)$ INV: $z = x(y_0 - y)$

while (y != 0) { VAR: y

$(| z = x \cdot (y_0 - y) \wedge 0 \leq y = E_0 |)$

$(| z + x = x(y_0 - y + 1) |)$ Impl $\wedge\, 0 \leq y - 1 < E_0 |)$

$z = z + x;$

$(| z = x(y_0 - (y - 1)) |)$ atrib $\wedge\, 0 \leq y - 1 < E_0 |)$

$y = y - 1;$

$(| z = x(y_0 - y) |)$ atrib $\wedge\, 0 \leq y < E_0 |)$

}

$(| z = x(y_0 - y) \wedge y = 0 |)$ ~~U~~ INV $\wedge \neg B$

$(| z = x y_0 |)$ Impl.