Equivalências 3/10 25

- $\neg \forall x \varphi \dashv\vdash \exists x \neg \varphi$
- $\neg \exists x \varphi \dashv\vdash \forall x \neg \varphi$

Se x não ocorre livre em ψ

- $\forall x \varphi \land \psi \dashv\vdash \forall x (\varphi \land \psi)$
- $\forall x \varphi \lor \psi \dashv\vdash \forall x (\varphi \lor \psi)$
- $\exists x \varphi \land \psi \dashv\vdash \exists x (\varphi \land \psi)$
- $\exists x \varphi \lor \psi \dashv\vdash \exists x (\varphi \lor \psi)$
- $\psi \to \forall x \varphi \dashv\vdash \forall (\psi \to \varphi)$
- $\forall x \varphi \to \psi \dashv\vdash \exists x (\varphi \to \psi)$
- $\forall x \varphi \land \forall x \psi \dashv\vdash \forall x (\varphi \land \psi)$
- $\exists x \varphi \lor \exists x \psi \dashv\vdash \exists x (\varphi \lor \psi)$
- $\forall x \forall y \varphi \dashv\vdash \forall y \forall x \varphi$
- $\exists x \exists y \varphi \dashv\vdash \exists y \exists x \varphi$

$\forall x\, P(x) \wedge Q(x)$

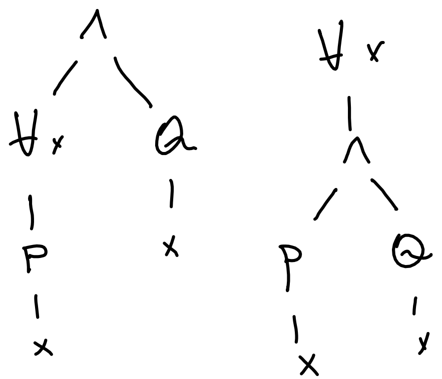
$\not\equiv$

$\forall x\, (P(y) \wedge Q(x))$

$\equiv$

$\forall x\, P(x) \wedge Q(y)$

$\equiv$ $\forall x\, (P(x) \wedge Q(y))$



Forma Prenex

$\forall x_1 \ldots \forall x_n \, \exists x_{n+1} \ldots \exists x_n \, (\underline{\quad})$

$\hookrightarrow$ matriz

$\forall x\, P(x) \wedge (\forall x\, Q(x) \rightarrow \exists y\, R(x, y, z))$

$\forall x\, P(x) \wedge (\neg \forall x\, Q(x) \vee \exists y\, R(y, y, z))$

$\forall y\, P(x) \wedge (\exists x \neg Q(x) \vee \exists y\, R(\overset{x_1}{x}, y, z))$

$\forall x\, P(x) \wedge (\exists x\, \exists y\, (\neg Q(x) \vee R(x_1, y, z))$

$\exists y\, (\forall x\, P(x) \wedge \exists x\, (\neg Q(x) \vee R(x_1, y, z)))$

$\exists y\, \forall x\, (P(x) \wedge \exists x\, (\neg Q(x) \vee R(x_1, y, z))$

$\exists y\, \forall x\, \exists x_2\, (P(x) \wedge (\neg Q(x_2) \vee R(x_1, y, z)))$

Exercício:

(1) Seja a altura de um termo definida como 1 + comprimento do caminho mais longo na sua árvore

Dados os símbolos de função d^0, f^2 e g^3, listar todos os termos sem variáveis de altura menor que 4.

$d \quad f(d, d)$

$f(f(d, d), d)$

② Desenhe a árvore de ~~(2-~~ ⌊28

$$(2 - S(x)) + (y * x)$$

③ a) Desenhe a árvore de φ

$$\neg(\forall x((\exists y P(x,y,z)) \land \forall z P(x,y,z)))$$

b) Indique as ocorrências de variáveis livres

c) Quais variáveis aparecem livres e não livres

d) Calcule $\varphi[t/x]$, $\varphi[t/y]$ e $\varphi[t/z]$ onde t é $g(f(g(y,y)),y)$

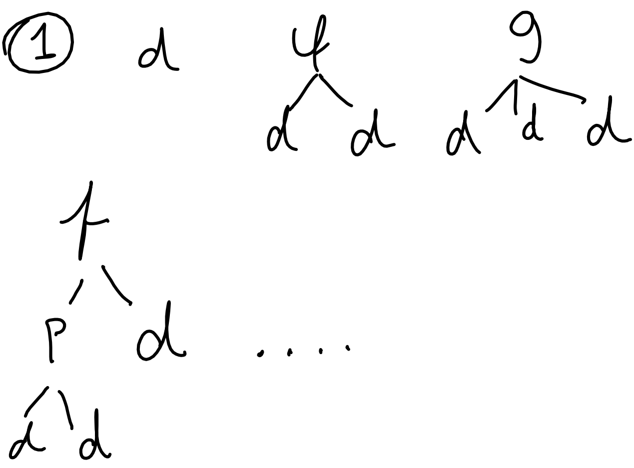
④ Mostre que os sequentes são válidos

ⓐ $\forall y P(x) \lor \forall x Q(x) \vdash \forall x(P(x) \lor Q(x))$

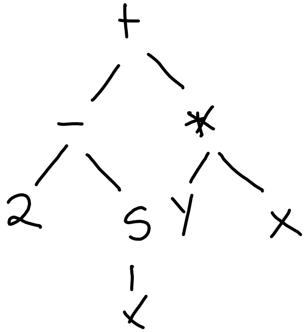
ⓑ $\neg \exists x P(x) \vdash \forall x \neg P(x)$

ⓒ $P(b) \vdash \forall x(x = b \rightarrow P(x))$

(1)



(2)



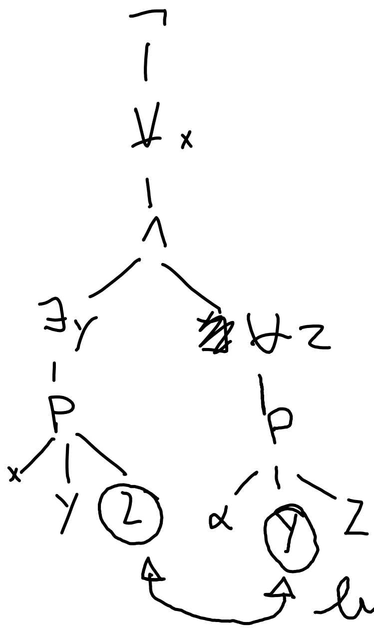
0

$s(s(0)) + (s(x), y) =$

$s(+(x, y))$

$s(0, x) = x$

(3)



livres

c, y, z

d) $\varphi[t/x] = \psi$,
pois ñ há ocorrência
livre

d) $\psi[t/y] =$ (30

$\neg(\forall x\,(\exists y\,P(x,y,z)) \wedge \forall z\,P(x, g(f(g(y,y)), y), z))$

$\psi[t/z] = \psi$, pois $y \in t$ não é livre

t nao livre p/ z em ψ

(4) a) $\forall x\,P(x) \vee \forall x\,Q(x)$

x_0

$\vdots$

$P(x_0) \vee Q(x_0)$

$\forall x\,(P(x) \vee Q(x))$

$$\frac{\forall x\,\varphi}{\varphi[t/x]}$$

1. $\forall x\,P(x) \vee \forall x\,Q(x)$
2. x_0

   $\forall x\,P(x)$ | $\forall x\,Q(x)$
3. $P(x_0)$ $\forall e$ | $Q(x_0)$
4. $P(x_0) \vee Q(x_0)$ $\vee i$ | $P(x_0) \vee Q(x_0)$
5. $P(x_0) \vee Q(x_0)$
6. $\forall x\,(P(x_0) \vee Q(x_0))$

b. 1. $\neg \exists x P(x)$ 31

2. x_0 $P(x_0)$ sup.

3.

4. $\exists x P(x)$ $\exists i$

5. $\bot$ $\neg e$

6. $\neg P(x_0)$ $\neg i$

$\forall x \neg P(x)$ $\forall i$

x_0

$\vdots$

$\neg i$

$\neg P(x_0)$

$\forall x \neg P(x)$

c 1. $P(b)$

2. x_0

$x_0 = b$ sup

$\vdots$

$b = x_0$

$P(x_0)$ $=e$

$x_0 = b \to P(x_0)$ $\to i$

$\forall x (x = b \to P(x))$

$$\frac{t_1 = t_2 \quad \varphi[t_1/x]}{\varphi[t_2/x]}$$

x_0

$x_0 = b$

$\vdots$

$P(x_0)$

$x_0 = b \to P(x_0)$

$\forall x (x = b) \to P(x)$