

Formais

1/10 (10)

- Variáveis livres
- Substituição $\varphi [t/x]$
 t livre para x em φ

$$\frac{\text{---} =_i}{t=t} \quad \frac{t_1 = t_2 \varphi [t_1/x]}{\varphi [t_2/x]} =_e$$

$$\frac{\forall x \varphi}{\varphi [t/x]} \quad \forall e \quad \boxed{\begin{matrix} x_0 \\ \vdots \\ \varphi [x_0/x] \end{matrix}} \quad \forall i$$

$$\forall x \varphi$$

$$\exists x \varphi$$

$$\varphi(x_1) \vee \varphi(x_2) \vee \dots \vee \varphi(x_n) \vee \dots$$

$$\frac{\varphi [t/x]}{\exists x \varphi} \quad \exists_i$$

x_0 é variável nova

$$\exists x \varphi \quad \boxed{\begin{matrix} x_0 \\ \varphi [x_0/x] \\ \vdots \\ x \end{matrix}} \quad \exists e$$

x

$$\forall x \varphi \vdash \exists x \varphi$$

177

1. $\forall x \varphi$

2. $\varphi[x/x] \forall e 1$

3. $\exists x \varphi \exists i 2$

$$\forall x (P(x) \rightarrow Q(x)), \exists x P(x) \vdash \exists x Q(x)$$

1. $\forall x (P(x) \rightarrow Q(x))$

2. $\exists x P(x)$

3. $x_0 P(x_0)$

4. $P(x_0) \rightarrow Q(x_0) \forall e 1$

5. $Q(x_0) \rightarrow_e 3, 4$

6. $\exists x Q(x) \exists i 5$

7. $\exists x Q(x) \exists_e 2, 3-6$

$$\forall x (Q(x) \rightarrow R(x)), \exists x (P(x) \wedge Q(x)) \vdash \exists x (P(x) \wedge R(x)) \quad (12)$$

$$1. \forall x (Q(x) \rightarrow R(x))$$

$$2. \exists x (P(x) \wedge Q(x))$$

$$3. x_0 P(x_0) \wedge Q(x_0)$$

$$4. Q(x_0) \rightarrow R(x_0) \quad \forall e_1$$

$$5. P(x_0) \quad \wedge e_1, 3$$

$$6. Q(x_0) \quad \wedge e_2, 3$$

$$7. R(x_0) \rightarrow e_6, 4$$

$$8. P(x_0) \wedge R(x_0) \quad \wedge i, 5, 7$$

$$9. \exists x (P(x) \wedge R(x)) \quad \exists i, 8$$

$$10. \exists x (P(x) \wedge R(x)) \quad \exists e, 2$$

$$\overline{\exists x P(x), \forall x \forall y (P(x) \rightarrow Q(y)) \vdash \forall y (Q(y))}$$

$$1. \exists x P(x)$$

$$2. \forall x \forall y (P(x) \rightarrow Q(y))$$

- 3. $x_0 P(x_0)$
- 4. $\forall y P(x_0) \rightarrow Q(y) \quad \forall e \ 2$
- 5. $P(y_0) \rightarrow Q(y_0) \quad \forall e \ 4$
- 6. $Q(y_0) \rightarrow_e \ 3, 5$
- 7. $Q(y_0) \quad \exists e \ 1, 3-6$

8. $\forall y Q(y)$

Equivalências / Quantificadores

$\forall x \forall y \varphi \dashv\vdash \forall y \forall x \varphi$

$\exists x \exists y \varphi \dashv\vdash \exists y \exists x \varphi$

~~$\forall x \exists y \varphi \dashv\vdash \exists y \forall x \varphi$~~

$\forall x \exists y (x < y) \dashv\vdash \exists y \forall x (x < y)$

$\neg \exists x \varphi \dashv\vdash \forall x \neg \varphi$

$\neg \forall x \varphi \dashv\vdash \exists x \neg \varphi$

$$\neg \forall x \varphi \vdash \exists x \neg \varphi$$

1. $\neg \forall x \varphi$

2. $\neg \exists x \neg \varphi$ (sup)

x_0

3. $\neg \varphi [x_0/x]$ sup

4. $\exists x \neg \varphi$ $\exists i$ 3

5. \perp

6. $\varphi [x_0/x]$ RAA 3-5

7. $\forall x \varphi$ $\forall i$ 3-6

8. \perp $\neg e$ 1, 7

9. $\exists x \neg \varphi$ RAA 2, 8

$$\exists x \neg \varphi \vdash \neg \forall x \varphi$$

1. $\exists x \neg \varphi$

2. $\forall x \varphi$ sup

3. $x_0 \neg \varphi [x_0/x]$

4. $\varphi [x_0/x] \forall e$ 2

~~\perp~~

5. \perp

6. \perp

7. $\neg \forall x \varphi$ $\neg i$ 2-6

$$\forall x \varphi \wedge \forall x \psi \dashv\vdash \forall x (\varphi \wedge \psi)$$

15

$$\exists x \varphi \vee \exists x \psi \dashv\vdash \exists x (\varphi \vee \psi)$$

Se x não ocorre livre em ψ :

$$- \forall x \varphi \wedge \psi \dashv\vdash \forall x (\varphi \wedge \psi)$$

$$- \forall x \varphi \vee \psi \dashv\vdash \forall x (\varphi \vee \psi)$$

$$- \exists x \varphi \wedge \psi \dashv\vdash \exists x (\varphi \wedge \psi)$$

$$- \exists x \varphi \vee \psi \dashv\vdash \exists x (\varphi \vee \psi)$$

$$- \varphi \rightarrow \forall x \varphi \dashv\vdash \forall x (\varphi \rightarrow \varphi)$$

$$- \forall x \varphi \rightarrow \psi \dashv\vdash \exists x (\varphi \rightarrow \psi)$$